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AXISYMMETRIC AND NON-AXISYMMETRIC TURBULENT DIFFUSION IN A PLAIN CIRCULAR TUBE AT HIGH SCHMIDT NUMBER

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Abstract—Measurements were made of the radial and tangential eddy diffusivities of mass in a fully developed turbulent flow in a plain circular tube for Schmidt numbers ranging from 760 to 1200 with Reynolds numbers ranging from 5230 to 23550.

The results, expressed as the ratio of the radial eddy diffusivity of mass to that of momentum and as the ratio of the tangential eddy diffusivity of mass to the radial, are found to be simple functions of non-dimensional radius. They are indistinguishable from some previous measurements at Prandtl and Schmidt numbers of about unity. This is in contrast with several theories of turbulent exchange which predict an effect of increasing Prandtl or Schmidt number on the ratios. It is concluded that both ratios are simple functions of non-dimensional radius and increasing Schmidt number and, probably, Prandtl number have no effect on them.

NOMENCLATURE

r, radius;
z, non-dimensional radius r/r_0.

Greek symbols
ε, eddy diffusivity;
ξ, ratio $\varepsilon_{h,r}/\varepsilon_m$ or $\varepsilon_{d,r}/\varepsilon_m$;
ψ, ratio $\varepsilon_{h,\omega}/\varepsilon_{h,r}$ or $\varepsilon_{d,\omega}/\varepsilon_{d,r}$.

Subscripts
d, mass;
h, heat;
m, momentum;
o, wall;
r, radial;
ω, tangential.

INTRODUCTION

IT HAS long been recognised that the Reynolds analogy which equates the radial eddy diffusivity of heat, $\varepsilon_{h,r}$, or mass, $\varepsilon_{d,r}$, with that of momentum ε_m has limited validity and that there is some effect of the molecular Prandtl or Schmidt number on the ratios $\varepsilon_{h,r}/\varepsilon_m$ and $\varepsilon_{d,r}/\varepsilon_m$. Some researchers have also suggested that there is some effect of Reynolds number and of distance from the tube wall on these ratios and several theories based on models of eddy behaviour have attempted to predict them. Thus Jenkins [1], Azer and Chao [2], Lykoudias and Toulukian [3] and Deissler [4] all used a model of a spherical eddy which exchanged heat and mass with its surroundings in some fashion and each theory predicts a Prandtl or Schmidt number effect on the ratios. On the other hand the simple assumption made by Petukhov [5] that the ratio was unity achieved good agreement with the experiments of Harriott and Hamilton [6] for Pr up to 10^5.

Sleicher [7], Abbrecht and Churchill [8] and, more recently, Quarmby and Anand [9] and Quarmby and Quirk [10] measured the ratios for a range of Reynolds number with Pr and Sc about unity. The results of the last two investigations are shown in Fig. 1. It appears that the results for both heat and mass transfer are the same and this common ratio ξ is a simple function of non-dimensional radius with no effect of Reynolds number.

Each of the experimental investigations mentioned shows $\xi(z)$ as greater than unity and increasing towards the wall; unfortunately, none of the theories agrees with this and only Azer and Chao suggest ξ is greater than unity for Sc or Pr about 0·7. The lack of agreement between the theories themselves is even more obvious at high Prandtl number.

Much interest has been shown recently in non-axisymmetric diffusion. Quarmby and Anand [11] and Quarmby and Quirk [10] measured the tangential eddy diffusivities of heat $\varepsilon_{h,\omega}$ and mass $\varepsilon_{d,\omega}$. They

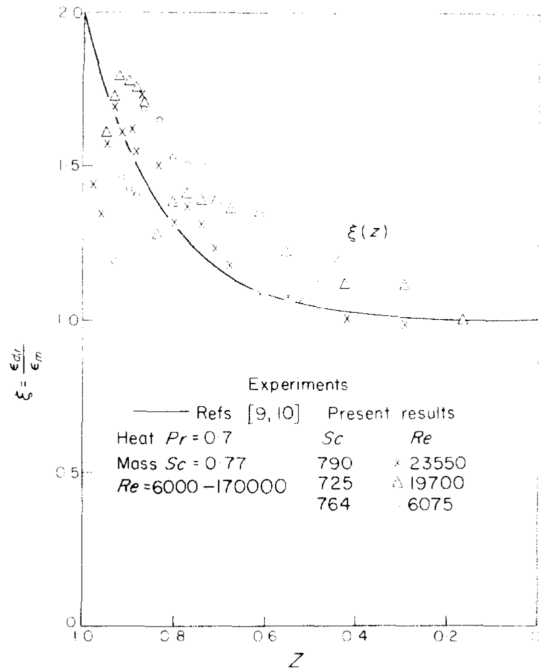

FIG. 1. Ratio of radial eddy diffusivities of heat and mass to that of momentum.

concluded that the two ratios were the same and this common ratio, ψ, was a simple function of non-dimensional radius. There was no Reynolds number effect.

It seemed very clear that measurements of the radial and tangential eddy diffusivities in fluids of high Prandtl or Schmidt number would be most useful. This work attempts such measurements. it complements the work of [9–11] for Prandtl and Schmidt numbers about unity.

EXPERIMENTAL INVESTIGATION AND RESULTS

(a) *Apparatus*

In the experimental investigation brine was diffused into water flowing turbulently in a plain circular tube. Mass transfer was preferred to heat transfer since some boundary conditions in such an experimental situation are precisely defineable. Also variations in fluid properties throughout the flow are difficult to make insignificant with heat transfer and high Prandtl numbers. For the purpose of determining any possible effects of ξ and ψ a high Schmidt number flow is as good as a high Prandtl number flow. The brine was diffused into the working section either from the bore of a sintered bronze disc, of the exact diameter of the working section and set flush in it, or from two small patches of Vyon porous plastic. The former gave an axisymmetric concentration profile and the latter a non-axisymmetric one.

Concentrations were measured by a specially made electrical conductivity probe. All measurements were carefully compensated for slight changes in temperature. The calibration curve and the effect on it of temperature were very carefully determined. The instrumentation was incorporated into a data logging system so that all readings could be put directly onto computer tape.

(b) *Axisymmetric diffusion results*

In the axisymmetric diffusion tests the Reynolds numbers were 6075, 19700 and 23550. The Schmidt number was calculated on the bulk mean concentration averaged over the measurements at each axial station.

Results for ξ were obtained from the measured developing concentration profiles using the data reduction techniques developed by [10]. They are compared in Fig. 1 with the earlier results for Prandtl or Schmidt number of about unity. Within the limits of experimental scatter which may be slightly greater in the present results there is no difference between the results for ξ with $725 < Sc < 790$ and those for $Pr = 0{\cdot}7$ and $Sc = 0{\cdot}77$. There is no apparent effect of Reynolds number.

(c) *Non-axisymmetric diffusion*

Preliminary tests established that the concentration profile was symmetric along any one diameter, that is, that the two patch sources were equal strength. Accordingly it was sufficient to make detailed measurements only between 0° and 90°. For each axial position the concentration was measured at about twenty radial positions and 5° intervals. Two Reynolds numbers were investigated 5320 and 21780.

The calculation of second derivatives were made by the methods of [10] and the results for ψ equal to $\varepsilon_{d,\omega}/\varepsilon_{d,r}$, where $\varepsilon_{d,r}$ is calculated from the results re-established here, are shown in Fig. 2. Also shown are the results of [10, 11] for $Pr = 0{\cdot}7$ and $Sc = 0{\cdot}77$. It can be seen that within the limits of experimental scatter the two sets agree and there appears to be no effect of increasing Schmidt number or of Reynolds number on the ratio.

DISCUSSION

The line of best fit, $\xi(z)$, for all the results shown on Fig. 1 is compared in Fig. 3 with the various theories already mentioned. The Prandtl or Schmidt number effect on the predicted ratio is shown where possible. It is not possible to give a result for very high Pr for

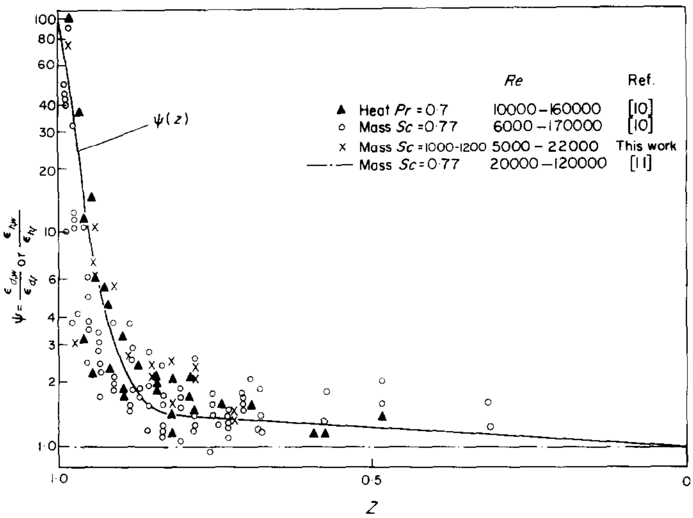

FIG. 2. Ratio of tangential eddy diffusivities of heat and mass to the radial.

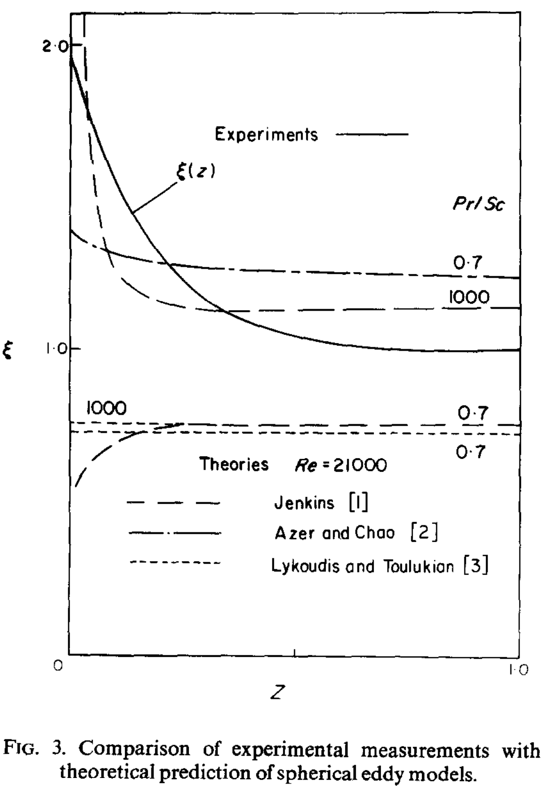

FIG. 3. Comparison of experimental measurements with theoretical prediction of spherical eddy models.

Azer and Chao since they confined their theory to $Pr < 15$, whilst Deissler's prediction for $Pr = 1000$ gives a ratio of about 3000 for the Reynolds number quoted. Jenkins theory shows a decrease with distance from the wall for high Pr which agrees with the present results but his prediction for $Pr = 0{\cdot}7$ is less than unity and shows an increase with distance from the wall. Lykoudis and Toulukian's theory shows no wall effect and almost no Prandtl number effect on the ratio. The lack of consistency between these theories was pointed out by Quarmby and Quirk in their discussion of their results for Pr or Sc of about unity. The present measurements for high Schmidt number reinforce the view expressed there that none of these theories is really correct. It is a matter for speculation that some of them have been used successfully to predict Nusselt numbers with high Prandtl number fluids. Such calculations are generally accepted as a good test of the validity of the assumptions concerning ξ, since, with a uniform wall flux boundary condition, say, the temperature or concentration profile and, thence the Nusselt number, are supposedly, quite sensitive to the assumptions used. In such cases the profile gradients are quite steep in the sublayer and the choice for $\xi(z)$ there is regarded as critical. In this work low Reynolds numbers were chosen to give the greatest amount of information about the behaviour of $\xi(z)$ in the sublayer. Possibly, the different methods of solution of the governing differential equation and the different descriptions of the eddy diffusivities and velocity profile lead in each case to a fortuitous combination which gives the correct result.

The authors are not aware of any published theories of turbulent exchange which predict the tangential eddy diffusivity ratio so no comparison can be made with the results shown in Fig. 2.

CONCLUSIONS

The present experiments for high Schmidt number flows show that there is no effect of increasing Schmidt number on either the radial eddy diffusivity ratio or the tangential eddy diffusivity ratio for fully developed flow in a plain tube. The ratios are simple functions of distance from the wall and, allowing for experimental error which is unavoidably greater in the present results, they are not distinguishable for earlier measurements for Prandtl and Schmidt numbers about unity. It seems that some well known theories of turbulent exchange are not correct in their prediction of an effect of high Prandtl or Schmidt number on the ratios. But this is not to say that there will not be an effect with Prandtl or Schmidt numbers much less than unity. The present results cast doubt on the validity of the spherical eddy model of turbulence used in the theories mentioned.

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DIFFUSION TURBULENTE AXISYMETRIQUE OU NON DANS UN TUBE CIRCULAIRE A GRAND NOMBRE DE SCHMIDT

Résumé—Les diffusivités turbulentes axiales et tangentielles de masse sont mesurées pour un écoulement turbulent établi dans un tube à section circulaire. Les nombres de Schmidt s'étendent de 760 à 1200 et les nombres de Reynolds de 5230 à 23550.

Le rapport de la diffusivité turbulente radiale de mass à la diffusivité turbulente radiale de quantité de mouvement et le rapport des diffusivités turbulentes massiques tangentielle et radiale, sont des fonctions simples du rayon réduit. Les résultats ne sont pas différents de ceux déjà connus et relatifs à des nombres de Prandtl et de Schmidt proches de l'unité. Ceci est en contradiction avec quelques théories de l'échange turbulent qui prédisent un effet croissant du nombre de Prandtl ou de Schmidt sur les rapports définis plus haut. On conclut que les deux rapports sont des fonctions simples du rayon réduit et que le nombre de Schmidt et probablement celui de Prandtl n'ont aucun effet sur eux.

ACHSENSYMMETRISCHER UND NICHT- ACHSENSYMMETRISCHER TURBULENZAUSTAUSCH IM GLATTEN KREISROHR BEI HOHEN SCHMIDT-ZAHLEN

Zusammenfassung—Es wurden Messungen gemacht über die radiale und axiale Verteilung des turbulenten Stoffaustauschkoeffizienten in einem glatten Kreisrohr bei vollentwickelter turbulenter Strömung für einen Schmidt-Zahl Bereich von 760 bis 1200 und Reynolds-Zahlen von 5230 bis 23550.

Es zeigt sich, dass die Ergebnisse, ausgedrückt als das Verhältnis von radialem Stoffaustauschkoeffizienten zu Impulsaustauschkoeffizienten und als Verhältnis von tangentialem zu radialem Stoffaustauschkoeffizienten, einfache Funktionen des dimensionslosen Radius sind. Sie sind nicht von früheren Messungen mit Prandtl-Zahlen und Schmidt-Zahlen bei Eins zu unterscheiden. Dies steht im Gegensatz zu einigen Theorien des turbulenten Austausches die einen Einfluss steigender Prandtl- oder Schmidt-Zahlen auf die Verhältnisse beschreiben. Der Schluss wurde gezogen, dass beide Verhältnisse einfache Funktionen des dimensionslosen Radius sind und dass wachsende Schmidt-Zahlen keinen Einfluss auf sie haben.

ОСЕСИММЕТРИЧНАЯ И НЕСИММЕТРИЧНАЯ ДИФФУЗИЯ В ПЛОСКОЙ КРУГЛОЙ ТРУБЕ ПРИ БОЛЬШИХ ЧИСЛАХ ШМИДТА

Аннотация—Приведены измерения радиальных и касательных коэффициентов вихревой диффузии массы в полностью развитом турбулентном потоке в плоской и круглой трубе при числах Шмидта от 760 до 1200 и числах Рейнольдса от 5230 до 23 550.

Установлено, что результаты, представленные в виде отношения радиальных коэффициентов вихревой диффузии массы к количеству движения и в виде отношения касательных коэффициентов вихревой диффузии массы к радиальным, являются простой зависимостью от безразмерного радиуса. Они не отличаются от некоторых предыдущих измерений при Pr и $Se \sim 1$. Это противоречит некоторым теориям турбулентного обмена, учитывающих влияние возрастающих чисел Pr и Se на эти отношения. Сделан вывод, что оба эти отношения предствляют собой простые зависимости от безразмерного радиуса и возрастающего числа Шмидта и, вероятно, не зависят от числа Прандтля.